A VORTICITY EQUATION FOR NUMERICAL LONG RANGE FORECASTING THE CORIOLIS PARAMETER IN THE SOLAR SYSTEM

John C. Freeman, Jill F. Hasling and Robert A. Stacy
Weather Research Center, Houston, Texas

ABSTRACT

Jule Charney, the father of numerical weather prediction, believed that the vorticity equation is the fundamental equation of meteorology.

$$\frac{d}{dt}\left(\frac{\zeta + f}{D}\right) = 0$$

Where: ζ = Vorticity of the Flow

f = Vorticity of the Earth

D = Thickness of the rotating fluid

He based his numerical weather prediction scheme on it. A correction to f, which has time scales that go from two weeks to 250 years, is introduced in this paper. This correction is likely to increase the accuracy in, and the use of, numerical prediction in long-range weather forecasting.

The authors believe that there is a rotation of the atmosphere, which is controlled by

– the orbit of the moon and Earth about their mutual center of gravity,

– the orbit of the Earth about the center of the Sun, and

– the orbit of the Sun about the center of gravity of the solar system.

The angular velocity of each of these bodies is a function of time. The periods of their orbits are 27.5 days, 1 year, and 178.8 years, respectively. The 178.8 year period has modulations of 11.9 years, 29.6 years, 83.7 years, 165.7 years and 247.7 years.

The long-term average of these orbits become part of the accepted value of the rotation of the Earth. However, the time-dependent portion of these rotations are new terms, which must be incorporated into the value of the rotation of the atmosphere to find the Coriolis parameter. The accumulated maximum value of these terms is 0.04 f; thus, it is not necessary to take them into account in a forecast for several days, but forecasts for a greater period than one month more would likely be affected.

The governing equation of long term, large-scale atmospheric patterns is the equation for conservation of absolute vorticity. This is the atmospheric relative vorticity plus the vorticity the atmosphere would have if there were no internal forces in the atmosphere to cause relative motion. If the vorticity changes because the rotation of the atmosphere changes, that change must be subtracted from the relative vorticity.

The majority of the rotations are common to the Sun and the Earth because they share the orbit of the Sun about the center of gravity of the solar system. This shared orbit is a subject of intensive investigation as the long-sought link between phenomena on the Sun and phenomena on the Earth.

Using a value for the vorticity of the earth, which is a function of a time, latitude, and longitude dependent Coriolis parameter is expected to improve the accuracy of long-range forecasts.

Weather Research Center, 3227 Audley, Houston, Texas, 77019

INTRODUCTION

One of the authors was an associate of Jule Charney while the foundation for numerical weather prediction was being developed. Charney was convinced that the direct use of the vorticity equation rather than the primitive equations should be used in numerical weather prediction. He believed that the vorticity equation was the fundamental equation of atmospheric motion and the inherent instability of the primitive equations would be impossible to overcome. He was convinced that the second idea was false when presented with a model by Freeman (1949), which contained Rossby waves and gravity waves and was capable of following both, if the time steps were small enough to maintain the stability of gravity waves.

Charney's first opinion may not be strictly correct because the primitive equations have been quite successful in making forecasts for seven to fifteen days. The fact remains that the vorticity equation is the controlling mechanism of large scale planetary flow. Recent work by Qin and van den Dool (1995) shows that the divergent vorticity model shows more skill after three to five days than more sophisticated primitive equation models. These considerations led the authors to further investigation of the divergent vorticity equation as a tool for long range forecasting, especially when new phenomena have been discovered which have long time scales.

The action of the vorticity equation needs to be clarified. It is a description of an instantaneous situation that describes a relationship between the vorticity that shows up on a map and the total vorticity of a fluid. It is best understood in the case where the vorticity, which appears on a map is zero. Then the total circulation is the circulation of the fluid because of its motion through space. For short-term numerical prediction, it has been assumed to be the rotation of the solid Earth once around in 24 hours. That is a good value for the solid Earth because all of the time variations in the terms are periodic with periods less than 10 or 15 thousand years and it would take that long for the time dependent terms to affect the massive Earth. This is not true, however, for the fluid envelope that separates the solid Earth from space. The gases and fluids in this envelope can respond immediately to changes in rotation. This paper illustrates that there are at least seven time-dependent rotations that affect the atmosphere at any time. Time dependence in the elliptic orbits of the moon and several planets is the mechanism. The total of the time dependent terms never exceeds 0.04 f so that the correction for time dependence need not be taken into account for forecasts of a few days. When the time frame is extended to a week or more it is expected to make a difference. Robert O. Reid (2004) suggests that the atmosphere, the space around the Earth is part of the solar system and not just the Earth; thus, the Earth's atmosphere has the capability of being affected by the Coriolis parameter of the solar system.

This paper is a result of twenty years of studying solar-terrestrial relationships. The authors have been inspired by a paper written by Clyde Stacey, in an encyclopedia edited by Rhodes .W. Fairbridge (1967). Stacey showed that the response of the Sun to Jupiter and Saturn led to an orbit about the center of gravity of the solar system and this orbit had the same period as the Sunspot cycle. Stacey's idea was adopted and emphasized by Fairbridge and the details of the orbit given by Jose (1965). To get a program that would reproduce Jose's results, a program giving Keppler's laws by Myles Standish (1989) of JPL was modified.

There are plenty of indications that the orbit has something to do with events on the Sun and Earth. The work of Jose (1965) related sunspots to the torque of the orbit. Landscheidt (1976) related the orbit to phenomena on the Earth and the Sun. The work of Freeman and Hasling conducted since 1985, shows similar relationships. The work by Freeman and Landscheidt was done without any knowledge of the shared orbits of the Earth and sun. It was based on the mountainous volume of work relating sunspots and the weather and the faith that if sunspots were related to phenomena on the Earth and sunspots were related to the orbit of the sun, then the events in the orbit of the sun should be related to phenomena on the earth. The revival of the knowledge of the shared orbit in 2003 gave added credence to this work

Hopfner (1999) showed that the QBO (Quarterly Biannual Oscillation) and ENSO (El Niño - Southern Oscillation) signals existed in the length of day records and that opens a question: Is the LOD (Length of Day) signal a result of the phenomena or is the signal in the LOD caused by the orbit which caused the phenomena.

A solid planet or other body, because of its enormous mass, has a tremendous amount of angular momentum in its rotation. The small effect due to the time dependent terms is not detectable in the period of the orbit of a planet. The fluid envelope, however, does not have this great angular momentum and can respond to the changes in angular velocity.

SHARED ORBITS

Newton's Law shows that the Earth is in orbit about the Sun and the Sun is in orbit about the center of mass of the solar system, but the fact that the Earth follows the motion of the Sun in it's orbit was not emphasized by Newton. Freeman and Hasling (2003) showed that this under emphasized fact is important in solar-terrestrial relationships. There is no detectable reference in the literature that states this fact plainly, but Standish (2003) of the Jet Propulsion Laboratory assures the authors of its truth, both in theory and observation.

The following discussion illustrates that the Earth is in orbit about the center of the Sun and the Sun is in orbit about the center of mass of the solar system; this leads to the conclusion that the Earth shares the orbit of the Sun, just as the moon shares the orbit of the Earth.

Imagine a star-system, made up of a star the size of the Sun and a planet, the size of the Earth, in a circular orbit about the star, 93 million miles in radius,. This orbit is about the center of mass of the star and planet system. The center of mass of the system is about 45 miles from the center of the star, a distance that is normally ignored and it is said that the planet orbits the star. The planet would take one year to go all the way around the star.

Now change this star-system by adding another star, the same size as the Sun. If this second star is placed in a orbit similar to that of Jupiter about the Sun, the two stars will orbit their center of mass (a point halfway between them). The planet would still make an orbit of one year as in the original system.

Next, reduce the mass of the star in "Jupiter's orbit" by a factor of five. This will bring the center of mass to a point on the orbit of the planet. The planet will still orbit the original star with a period of one year.

Finally, arrange the bodies into a solar system, similar to ours but with only the

Sun, Earth, and Jupiter. The center of mass would be located just outside the Sun. The Sun and Jupiter both orbit the center of mass of the system, and the Earth is orbiting a point that is approximately the center of the Sun. Thus, the Earth shares the orbit of the Sun about the center of mass of the Sun-Jupiter system.

The Earth's rotation has a term that is a fossil of the planets' formation and very small terms that result from global average winds and ocean currents but many moderate terms that result from various orbits. Thus, it is appropriate to investigate orbits and how they combine.

BIORBS, THE GENERAL TWO-BODY ORBIT

The fundamental unit of the solar system is the two-body orbit. Something like an orbit must exist for two bodies to remain near each other and not collide. The term "body" represents a spherical piece of matter of any mass. If there were only two masses in the universe, a distance D apart, and each body was stationary, then the gravitational attraction would move the masses toward each other and they would eventually collide and become one body. A body of mass M, traveling at speed V, will stay a distance R from a fixed point C if force MV^2/R is pushing it toward C at all times. Gravitational attraction between the two bodies results in an attraction of each of them of $G(M_1)(M_2)/R^2$ toward the other. If the primary body has mass M_1 and is a distance R_1 from a stationary center and the secondary body, has a mass M_2 is a distance $R_2 = (R - R_1)$ from the center then it is possible, if $M_1R_1 = R_2\,M_2$, for the body M_1 to be circling the center in an of radius R_1 and the body M_2 on the opposite side of the center circling the center in an orbit of radius R_2. The resulting paradigm is a special case of the two-body orbit.

The general two body orbit is a fundamental building block of the solar system, the authors have chosen to call it a "biorb". A biorb consists of a Primary body (P) and a Secondary body (S), a distance R apart. If P and S are distances R_P and R_S, respectively, from the Center of Mass (CM) of the (P&S) biorb, then $R_P + R_S = R$. For two elliptical orbits that are geometrically similar, CM is the focus of two ellipses. P and S's angular velocities and periods are equal at every instant and the angle of the rotation vector is fixed for all time. The least distances to each ellipse are in opposite directions, as are the greatest distances. The revolving planets are always in corresponding positions on their orbits so that they keep a constant ratio of distances R_P/R_S. A biorb is illustrated in Figure 1.

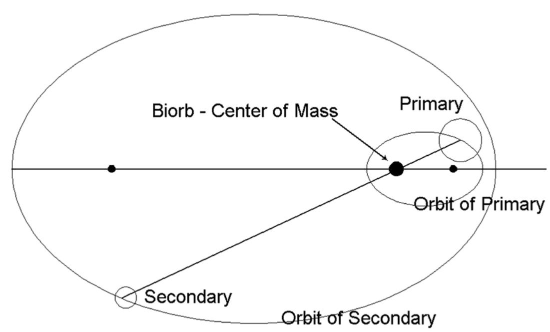

Figure 1. Illustration of a Biorb.

To obtain the motion resulting from two planets revolving around a star, first construct Biorb 1 and Biorb 2, as shown in Figures 2a and 2b, respectively.

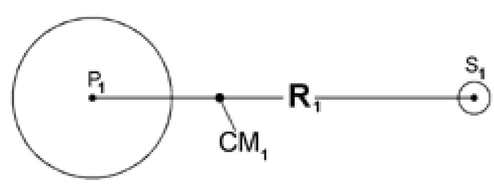

Figure 2a. Biorb 1.

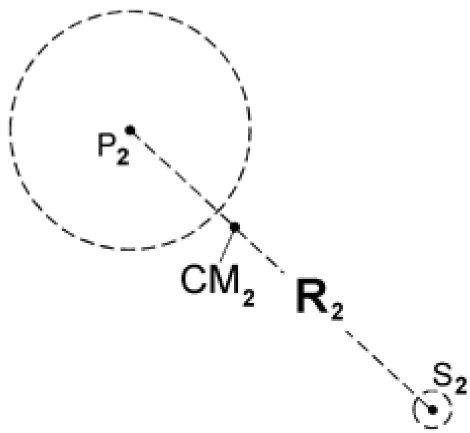

Figure 2b. Biorb 2.

Now consider the combined biorb for Biorbs 1 and 2. If the center of P_1 is placed on the center of P_2, then the center of mass, $CM_{1,2}$ of the two planet system is the vector addition of vector $\overline{P_1\ CM_1}$ and $\overline{P_2\ CM_2}$. This combined biorb is shown in figure 3.

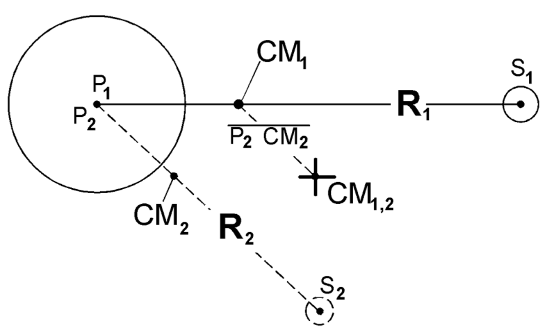

Figure 3. Biorb 1,2.

The angles, the period, and the shape of Biorbs 1 and 2 remain unchanged.

$P_{1,2}$ is in orbit about $CM_{1,2}$. As shown in Figure 4., at any instant in time, S_1 , and S_2 are translated in the same direction and at the same speed of $P_{1,2}$. This is in addition to their normal orbital motions.

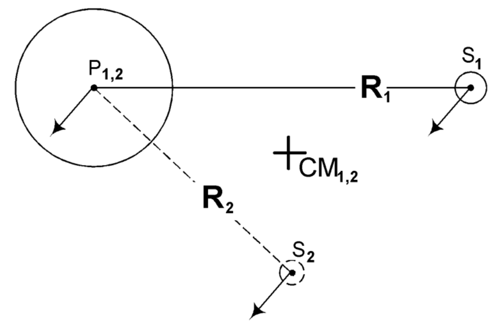

Figure 4. Instantaneous velocity of the combined biorb

OUR SOLAR SYSTEM

Another kind of complex biorb is made up of a star and a biorb. In this case, the center of mass of the biorb is in orbit and takes the place of the center of a planet the Biorb 2 keeps all of its properties but position. If you apply these procedures to the solar system then they should first be applied to the planets and their moons to get nine complex biorbs that are combined as above to get our solar system. If this procedure is followed, the Sun moves through an orbit that is about a million miles in radius and the Earth's orbit is centered on a point about fifty miles from the center of the Sun and follows the orbit of the Sun about the center of the solar system. This is proof that the Sun orbits the center of the solar system while the Earth orbits the center of the Sun.

As a first step, the authors have investigated the effect of the 5 outer planets (Jupiter, Saturn, Neptune, Uranus, and Pluto), the Earth, and the Moon. The next phase of the work will include the angular velocities of the three inner planets (Mercury, Venus, and Mars), the asteroid belt, and the newly discovered planet, Sedna.

The orbits that affect the atmosphere are the orbit of the Earth about the center of the Sun and the orbits of five outer planets (because the Earth follows the center of the Sun.) The weather of the Earth is affected by time variations in seven different elliptical orbits. The time variations average out to zero over 2000 to 3000 years, the time it takes for an orbit to change the angular velocity of the solid Earth so that it remains constant throughout the process.

The inclusion of the time variations in the basic velocity adds new terms to the vorticity equation that have periods of 28 days to 250 years. This forms the physical basis for long-term changes in the general circulation of the Earth’s atmosphere furnish the link between Sunspot activity and the Earth’s weather.

The Sun’s orbit is not as stable as the individual orbits of the planets but almost repeats itself every 178.7 years. Figure 5 shows this instability of the Sun’s orbital path.

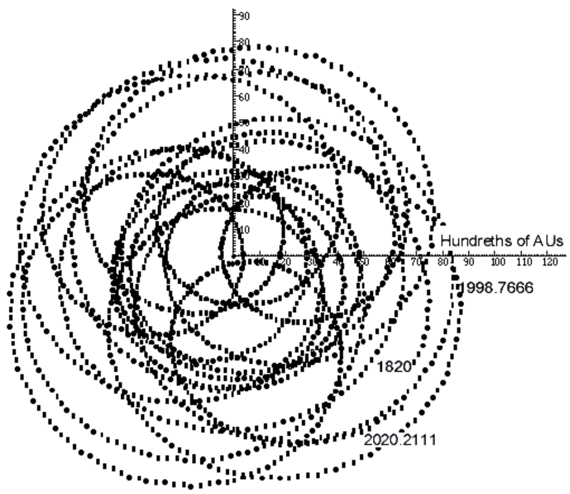

Figure 5: Projection of the 178.8 Year Period of the Sun on the Invariant Plane.

This diagram represents one completion of the Sun’s 178.7 year orbit through the solar system. The authors have named this 178.7 year orbital period of the Sun a “helio-epoch” and it can be seen in the diagram that one circle represents one helio-epoch. From Figure 1 notice the variance in the sixteen circuits of the Sun about the center of gravity of the solar system that are graphed. This is evidence that the Sun’s orbit, or helio-epoch, is variable.

Jose [1965] developed a graph when he made use of computers to compare the motion over several hundred years of all the planets and the Sun. The Sun’s orbit ranges from 0 to 1.5 million miles where the radii of orbits of the outer planets range from 480 million to 3,670 million miles. Therefore, the radius of the Sun’s orbit is much smaller than the radii of the orbits of the five outer planets.

The orbit of the Sun, about the center of gravity of the solar system, is shared by the Earth; like the moon shares the orbit of the Earth about the Sun as was shown by Freeman and Hasling (2003). The principal method by which this sharing takes place is explored in the present paper. Each of the shared orbits has an angular velocity that shows up in the vorticity equation for the body being studied.

DEVELOPMENT OF THE CORIOLIS PARAMETER

Coriolis (1835) determined that if a spaceship were accelerating in a curved path through the solar system that any motion studied in the spaceship must take the change in speed and the angular velocity of the spaceship into account. If this spaceship were moving in the same direction as the center of a planet or star this fact would still be true. Hence, the motion of the center of a planet or a star can be calculated from all of the orbits that cause significant motions of that body.

In general an orbit is the path of a body in a plane. It is an ellipse and it has a period, T, which is the time between subsequent passes of the same point of the ellipse. Its angular velocity varies with the distance from the center of rotation, which forms one focus of the ellipse. However, the instantaneous angular velocity is a variable. The angular velocities have the relationship

$$\Omega(t) = \Omega_a + \omega(t)$$

where $\omega(t)$ is the time variation of the angular velocity. Notice that when the angular velocity is averaged over one period that $\omega_a(t) = 0$. The average angular velocity, Ω_a, the normal direction to the plane of the orbit and the period are all constant.

To find the total effect of several orbits on one body the vector angular velocities are added, so that

$$\Omega_o(t) = \Omega_{oa} + \omega_o(t).$$

Now the body has its own fossil constant angular velocity left over from its formation, Ω_f, so that its total angular velocity is

$$\Omega_T(t) = \Omega_f + \Omega_{oa} + \omega_o(t)$$

The two constant values are combined:

$$\Omega_T(t) = \Omega_{Ta} + \omega_o(t).$$

Two examples from our solar system follow.

<u>Example1</u>:

The Sun is in orbit with the five outer planets. The orbital angular velocity of the Sun is

$$\Omega_{os}(t) = \Omega_j(t) + \Omega_s(t) + \Omega_u(t) + \Omega_n(t) + \Omega_p(t);$$

or

$$\begin{aligned}\Omega_{os}(t) &= \Omega_{ja} + \Omega_{sa} + \Omega_{ua} + \Omega_{na} + \Omega_{pa} \\ &\quad + \omega_j(t) + \omega_s(t) + \omega_u(t) + \omega_n(t) \\ &\quad + \omega_p(t) \\ &= \Omega_{osa} + \omega_{os}(t).\end{aligned}$$

If Ω_{sf} is the fossil value for the Sun and $\Omega_{ss}(t)$ is the total angular velocity, then

$$\Omega_{sso}(t) = \Omega_{sf} + \Omega_{osa}.$$

Ω_{ssa} is the fossil value plus the average of the angular velocities of the orbits; then

$$\Omega_{ss}(t) = \Omega_{sf} + \Omega_{os}(t)$$

This is the angular velocity that is measured and used for the Sun.

$$\begin{aligned}\Omega_{ss}(t) &= \Omega_{ssa} + \omega_j(t) + \omega_s(t) + \omega_u(t) + \\ &\quad \omega_n(t) + \omega_p(t) \\ &= \Omega_{ssa} + \omega_{ss}(t)\end{aligned}$$

The five outer planets and the Sun are in orbit about the center of gravity of the solar system. $\Omega_{ss}(t)$ must be taken into account when calculating the angular velocity of the Earth.

<u>Example 2</u>:

The effect of the orbits of the Earth about the Sun and the moon about the Earth are added to the effect of the orbit of the Sun on the Earth to get the formula:

$$\Omega_{eo}(t) = \Omega_{os}(t) + \Omega_{mo}(t) + \Omega_{es}(t)$$

$\Omega_s(t)$ is the angular velocity of the orbit of the Earth about the Sun. $\Omega_{mo}(t)$ is the angular velocity of the moon about the Earth, which is the same as the angular velocity of the Earth about the center of gravity of the Earth-moon system, which is a significant motion.

$$\begin{aligned}\Omega_{eo}(t) &= \Omega_{osa} + \Omega_{moa} + \Omega_{esa} + \omega_{ss}(t) + \\ &\quad \omega_{mo}(t) + \omega_{es}(t)\end{aligned}$$

Add the fossil value Ω_{fe} to this and get the value of the rotation of the Earth, which is

$$\Omega_{ee}(t) = \Omega_{eea} + \omega_{ee}(t)$$

$\Omega_{eea} = 2\pi$/day and it is the value of Ω that is accepted by scientists.

$$\begin{aligned}\omega_{ee}(t) &= \omega_j(t) + \omega_s(t) + \omega_u(t) + \omega_n(t) + \omega_p(t) \\ &\quad + \omega_{mo}(t) + \omega_{es}(t)\end{aligned}$$

So the angular velocity of the Earth has seven orbital terms. Those are due to Jupiter, Saturn, Uranus Neptune, Pluto, the orbit of the Earth about the Sun and the orbit of the moon about the Earth.

The Coriolis parameter $2\Omega_{eea} \sin \phi = f$ (where ϕ is the latitude) is the only term of the absolute vorticity that is used at the present time in equations of geophysics. It will now be shown that $2\,\Omega_{ee}(t) \sin \phi = f(t)$

should be used instead.

A portrayal of the Coriolis parameter is shown in Figure 6a.

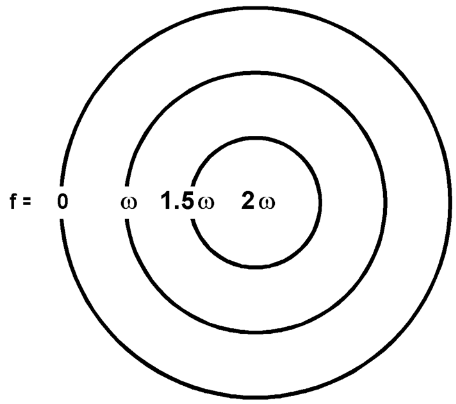

Figure 6a.

Figure 6b shows Figure 6a overlayed with:

$$f(t) = 2\,\dot{\omega}(t)\,\sin\phi,$$

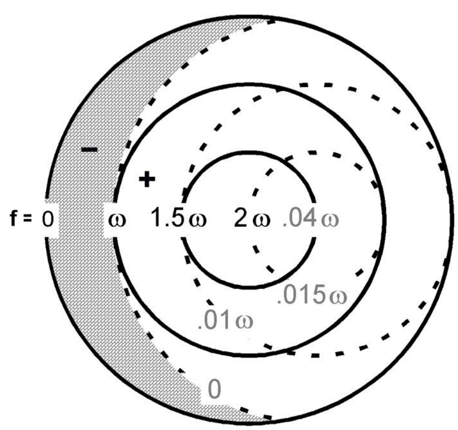

Figure 6b.

the time dependent value at an instant in time, which should be added to f. This shows what the proposed change in $\dot{\Omega}$ would be like.

Looking down on the pole, f = 2ω at the pole and steadily decreases to 0 at the equator. The value of f(t) at an instance in time of 0.02f is added and its center is displaced from the pole. The center stays still in the solar system as the Earth rotates under it. As indicated by the "-" sign, in the shaded area, the f(t) is subtracted from f.

The simplicity of the discussion evaporates when the problem of expressing the vertical component of the atmosphere's vorticity at a place on the Earth but is a function of time and position and the vorticity of the flow is added to get the vorticity equation. This new vorticity equation for the atmosphere has terms involving the longitude and time that must be taken into account in calculating the forecast relative vorticity.

CONCLUSIONS

This equation could lead to many new discoveries in geophysics. Some topics where new discoveries are expected are solar-terrestrial relationships, extended range forecasting, seasonal outlooks, Madden-Julian oscillation, quasi-biennial oscillation, ENSO events and other atmospheric phenomena. In addition topics in space weather, ionospheric circulation, oceanography, motion in the Earth's core, planetary science and solar physics.

There are many experiments that would indicate the truth of this theory. Two of them are proposed by J. Qin and Huug van den Dool (1995) of CPC have a barotropic model for the global atmosphere that they run in real time to make 15-day forecasts. After four or five days their model shows more skill than more sophisticated models. One experiment would be to add f(t) to the vorticity equation in their model and see if the skill increases. A second experiment would be to start with the normal annual barotropic flow and perturb it with the new vorticity equation and see if the resulting flow shows any signs of ENSO, QBO, Madden-Julian or other circulation patterns that existed in the time covered.

REFERENCES

de Coriolis, G. G., 1835: Sur les Equations Du Mouvement Relatif des Sustes De Corps. *J. Éc. Roy. Polvt. (Paris)*:, **15**:142-154.

Fairbridge, Rhodes W., 1967: *The Encyclopedia of Atmospheric Sciences and Astrogeology*, Reinhold Publishing Corporation.

Freeman, J. C., 1949: A Model that Displays Rossby Waves and Gravity Waves, Unpublished Research Paper, Institute for Advanced Study.

Freeman, J. C., and J. F. Hasling, 2002: The Effect Of The Sun's Orbit On The Earth's Atmosphere (poster paper), *83rd Annual Meeting of the American Meteorological Society*, Long Beach, California.

Freeman, J. C. and J. F. Hasling, 2003: An Orbital Motion Showed by Sun and Earth Affecting sunspots and Earth Weather, submitted to the AGU.

Hopfner, Joachim, 1999: Interannual Variation in Length of Day and Atmospheric Angular Momentum With Respect to ENSO Cycles, *22nd General Assembly, International Union of Geodesy and Geophysics*, Birmingham, UK, 12-13 July, 1999.

Jose, P. D., Sun's Motion and sunspots, 1965: *Astronomical Journal* **10**, (1), 193-200.

Landscheidt, T. 1976: Beziehungen zwischen der Sonnenaktvitat und dem Massenzentrum des Sonnensystems, *Nachrichten der Olbers-Gessellschaft 100.*

Landscheidt, T., 1983: Solar Oscillations, Sunspot Cycles, And Climatic Change, *Weather And Climate Responses To Solar Variations*, B.M. McCormac, Associated University Press, Boulder, 293-308.

Qin J., and H. M. Van Den Dool, 1995: Simple Extensions of and NWP Model, *Monthly Weather Review*, **12.**

Reid, R. O., 2004: Personal Communication, Texas A&M University.

Standish, E. M., 1989: Personal Communication, Jet Propulsion Laboratory.

Standish, E. M., 2003: Personal Communication, Jet Propulsion Laboratory.